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ABSTRACT

This book is a collection of teaching strategies and activities for teachers of secondary mathematics. This volume is the product of a workshop that focused on student understanding of directed numbers. Suggested teaching methods include introducing the number concept, using a number line, number strips, monograms, bottle top addition and subtraction, patterns, the hare and the hounds, win or lose, magic squares, null words puzzles, multiplication matrix, using a calculator, and games. Areas of mathematics which involve the use of directed numbers and students' problems with these concepts are also discussed. (DDR)

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Teaching Directed Numbers at Secondary School Level

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NOTES FROM A WORKSHOP ON

DIRECTED NUMBERS

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- British Development Division (BDDC) for providing the funding for this series of publications

- Volunteers and local colleagues contributing to production of publications.

- Organisation of Caribbean Overseas Development (OCOD) for assisting in the reproduction of these publications

Other publications in this series include:

Maths and Science

1. A Practical Workbook for CXC Biology
2. Data Analysis Questions for Science Subjects. A Resource Booklet
3. Exercises and Activities in Basic Number Work
4. Fractions. Activities and Exercises for Teaching Fractions in Secondary Schools
5. Lower School Maths. Lesson Plans and Activities for Ages 7 -9 Years.
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Many of these publications derive from projects or workshops funded through **VSO's Community Project Scheme** - an initiative also funded by grant from British Development Division, Caribbean.

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INTRODUCTION

Secondary School Teachers working in North Leeward, St Vincent were concerned about the problems which pupils encountered whilst working with positive and negative numbers. It was decided that our second workshop would have directed numbers as its theme. The details of this workshop are outlined below:

Teaching Directed numbers

Before attending the workshop participants are asked to consider the following:

1. Areas in Mathematics which involve the use (and understanding) of Directed Numbers.

2. The problems which students encounter in trying to understand Directed Numbers.

3. The problems which you as a teacher have encountered trying to teach Directed Numbers.

4. Teaching strategies which you have tried - (either successfully or unsuccessfully).

5. Any solutions you may have found in helping pupils overcome their difficulties.

During the workshop we plan to discuss each of the above items and hopefully devise some strategies which we can try with our pupils.
N.B. The V.S.O.'s do not have any solutions to this problem and are keen to exchange ideas and learn from this workshop.

Please bring relevant text books and any other resources you may have used plus examples of pupil's work - this may give some pointers as to the exact problems they encounter.

This booklet was assembled following the workshop and covers the main areas which were discussed:

1) topics which require the use of directed numbers.

2) The problems which pupils and teachers encounter when learning / teaching Directed Numbers.

3) Some exercises which could be used when teaching directed numbers.

We hope that these exercises will prove to be a useful resource to teachers.

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Maths Topics which involve

the use of Directed Numbers

Directed Numbers is a topic in itself and students must become proficient in manipulating positive and negative numbers in all four rules.

An understanding is also necessary if students are to understand the following -

1) Manipulating equations of all types.

2) Co-ordinates and graph plotting.

3) Algebra: factorising and rearranging formulae.

4) Transformations and Vectors.

5) Functions and Inequalities.....

It was recognized that a firm basis should be in place during Forms 1 - 3 if students were to fully understand the topics in Forms 4 - 5 on the CXC Basic and General papers.

Suggested Teaching

Methods

Problems encountered by Pupils and Teachers

As a result of much discussion the following is a list of some of the problems which may contribute to a student's lack of understanding of the concepts involved in manipulating Directed Numbers.

1) Students have very few real life experiences of negative values e.g. they do not experience temperatures of -10^{0} - although they may know how cold the inside of a refrigerator is.

2) We rarely write positive fifteen as +15 -this is a new concept which students must become familiar with when handling Directed Numbers.

3) Notation causes problems, i.e. when does '−' mean add and when does it mean a positive value?

4) Students experience problems in ordering negative numbers and are unable to say which is the largest number if they are given -5 and -8.

5) Students usually have more problems handling addition and subtraction of positive and negative numbers - multiplication and division generally cause less confusion. Students often confuse the rules and will approach a question such as

$$(-3)-(-5)$$

and say 'two negatives make a positive, therefore the answer is − 8.'

Introducing any Mathematical concept should, where possible relate to daily life and experiences. Some ideas include:

1) Climbing up a tree / Volcano
Diving below sea- level

2) Climbing a tree / ladder
Digging a hole / well

3) Having money
Owing money

4) Temperature outside in the sun
Temperature inside a refrigerator / freezer

5) Peugeot, a card game played in St. Vincent which involves adding and subtracting points.

Use diagrams to illustrate the above examples and discuss the concepts of above / below introducing the concept of positive and negative values.

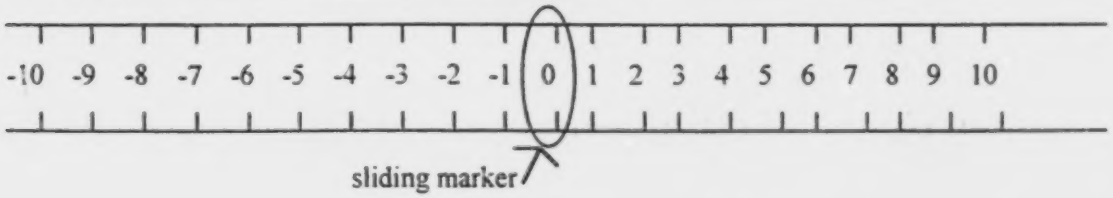
Extension

Let students work in groups and draw a picture to represent one of the above situations and then devise questions relevant to that problem. Present to the rest of the class. Alternatively, this could be given as a homework -display the results on the classroom wall.

Using a Number Line

Although students often resent having to make and use a number line (considering it to be rather too simple a learning aid) it is one of the most visual tools which students can use.

Make a number line showing positive and negative numbers. Use strong card and large numbers and display the 'line' on the blackboard or classroom wall. Use a large paper clip or circle of coloured paper to mark a position on the line which shows the result of a movement along the line. Make the 'line' as long as possible.



Teaching Strategy

1) Starting at zero, introduce the concept of walking forward 5 metres then back 6 metres - where do you finish?

2) Repeat (1) using the idea of putting money into the bank (having) and taking money out (owing).

3) Use plenty of examples from (1) and (2) . (Use appropriate and available text books for exercises).

4) Develop the concept of:

   (a) adding a positive number - move to the right

   (b) subtracting a positive number - move to the left

   (c) adding a negative number - move to the left

   (d) subtracting a negative number - move to the right.

   (Again use text books for exercises)

5) Use the number line to discuss the order and size of the numbers - the largest number in a group lies to the right on the number line.

N.B. Students should make their own number lines to use as reference.

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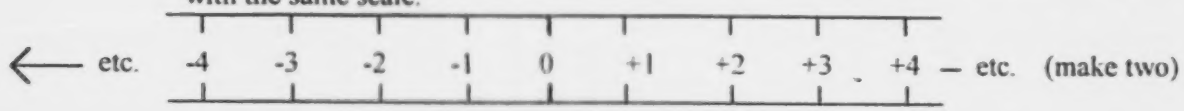
NUMBER STRIPS

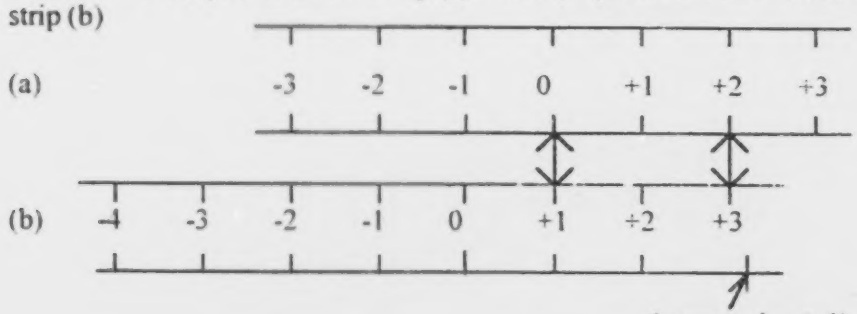
1) You will need to make two number strips, large enough for use in the classroom, both with the same scale.



(make two)

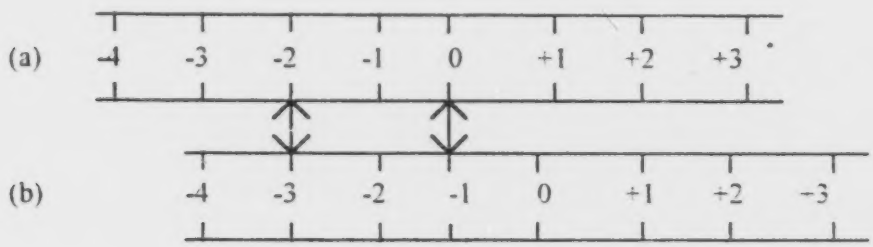
Student should make their own copies.

2) One strip (a) remains fixed (i.e. pin it to the board or desk) and one strip (b) slides. To add (+1) + (+2), place the 0 on strip (a) above (+1) and read the answer below (+2) on strip (b)



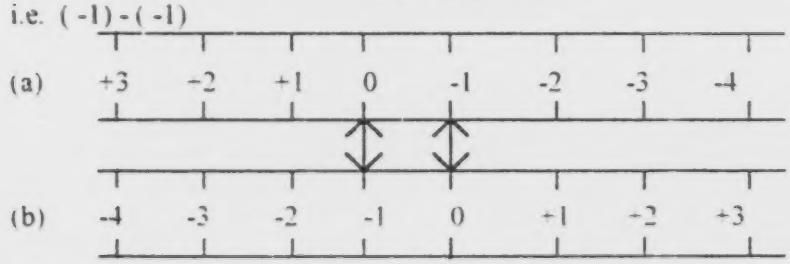
Answer : i.e. (+1) + (+2) = (+3)

To add (-1) + (-2) place the zero on strip (a) above the (-1) on strip (b) and read the answer below (-2)



Answer : i.e. (-1) − (-2) = (-3)

3) To subtract two numbers use strip (a) upside down.
i.e. (-1) - (-1)



Answer : i.e. (-1) - (-1) = 0

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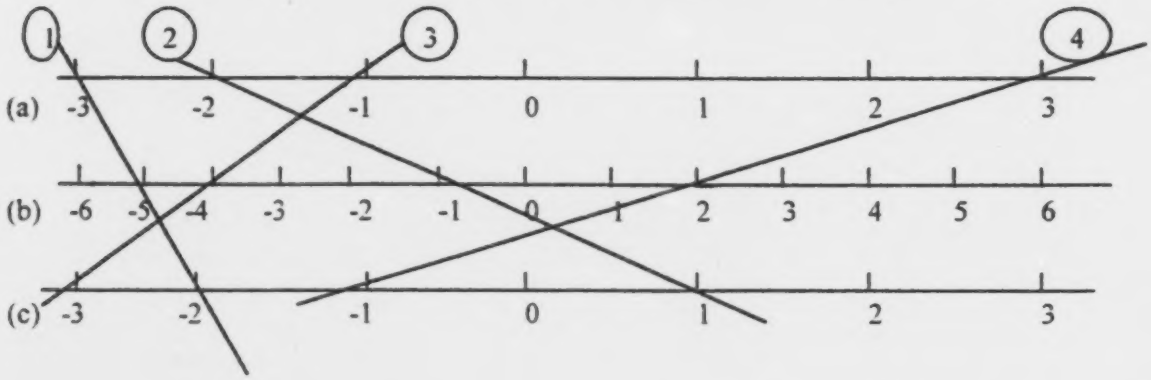
NOMOGRAMS

This is another visual aid to adding and subtracting with positive and negative numbers. Draw 3 number lines (which all remain in the same fixed position).



To add two numbers together, find the numbers on lines (a) and (c) , join with a ruler, read the answer from line (b).

e.g. (1) (-3) + (-2) = -5 (3) (-1) + (-3) = -4

(2) (- 2) + (+ 1) = - 1 (4) (+3) + (-1) = +2

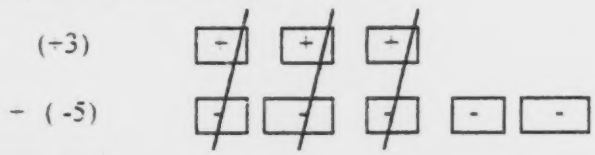
To subtract two numbers read the first number from line (b) , the second number from line (a) and the answer from line (c).

USING BOTTLE TOPS OR COUNTERS TO HELP ADDITION AND SUBTRACTION.

1) You will need plenty of bottle tops (10 - 15 per student) or use squares of paper marked '+' on one side and '-' on the other side.

2) To demonstrate addition and subtraction:

(a) (+3) - (-5): place squares of paper on the desk :-



remove [-] and [-] pairs ⟶ answer = -2

NB (-3) - (-5) follows the same rule.

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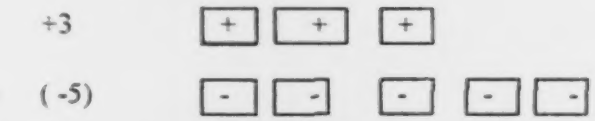
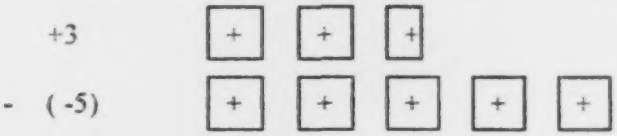
(b) (+3) - (-5) : place squares as follows:



but because we are subtracting turn each of the negative cards over to show



count the positive signs ⟶ answer = +8

(c) (-3) - (-5) would result in:

remove the [-] and [-] pairs as in (a) ⟶ answer = +2

NB. If using bottle tops allocate one surface as [+] and one as [-].

3) Practice by giving plenty of examples (use a suitable text book), making sure the students write out the questions and the answers so that they have questions to refer to.

PATTERNS

Some students may find it useful to look for patterns in addition and subtraction e.g. Copy and complete:

1)
$(+5) + (+3) = +8$
$(+5) + (+2) =$
$(+5) + (+1) = +6$
$(+5) + 0 =$
$(+5) + (-1) =$
$(+5) + (-2) =$
$(+5) + (-3) = +2$
$(+5) + (-4) =$
$(+5) + (-5) = 0$
$(+5) + (-6) =$
$(+5) + (-7) =$
$(+5) + (-8) =$

2)
$(+5) - (+3) = +2$
$(+5) - (+2) =$
$(+5) - (+1) = +4$
etc.
$(+5) - (-8) = +13$

3)
$(-5) - (+3) = -8$
$(-5) - (+2) =$
$(-5) - (+1) =$
etc.
$(-5) - (-8) = +3$

4)
$(-5) - (-3) = -2$
$(-5) - (-2) =$
$(-5) - (-1) =$
etc.
$(-5) - (+8) = -13$

Repeat with multiplication using both positive and negative signs.

NB. Some text books use subscript notation to distinguish the sign of a number from the operation i.e. $^{+}5 - ^{-}3$. Teachers may like to consider using this method, for example, with the above patterns.

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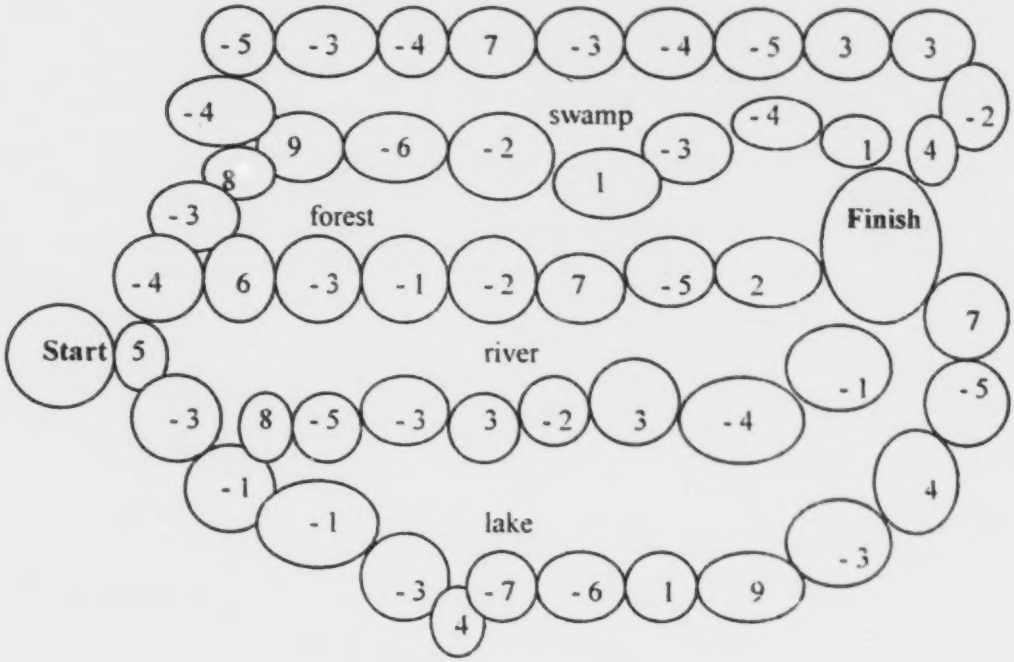
In the maze below there are several parts which lead from start to finish. You can travel from one circle to the another only by traveling through the boundary of two circles which touch ! The sum of the numbers in each of the circles along any path gives the total for that path. The hare has traveled along the path whose total is zero. Can you find the path taken by the hare?



WIN OR LOSE ?

This table shows the amount of money which was won or lost each day.

		Net win or loss
Monday	12 - 6 + 14 - 11 - 3	
Tuesday	15 - 9 - 11 - 14 - 2	
Wednesday	- 21 + 6 + 9 + 32 - 17	
Thursday	- 12 + 7 - 21 + 29 + 16	
Friday	14 + 12 - 6 - 31 + 27	
Saturday	56 - 32 + 76 - 29 - 41	
	Win or loss for the week	

1) Work out the amount won or loss each day and write the answer in the third column.

2) Work out the amount won or loss for the week and write in the last box in column 3.

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Copy and complete the magic squares . The sum of the numbers in each row, column or diagonal should always be the same.

1)

	-3	2
		3
		-2

2)

5		3
	2	
1		

3)

	1	-4
	-3	
-2		

4)

	-4	1
	0	
	4	

5)

-2		
-7	-5	-3

6)

		2
-4	1	0

7)

	7		9
			6
5	-2	11	0
	1	4	

8)

-6			6
	-1		
7	0	4	
-3		-2	9

9)

	-9		5
		-1	-6
1		-5	-2
	3		-7

10)

7		-5	-2
-4			
6		-6	-1
-7			4

11)

0			-3
		-6	-8
-7	-9	-10	-4
		-1	

12)

7			-5
		1	8
	4	0	
10	-1		-2

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NULL WORDS PUZZLES

Purpose: To practice the addition of positive and negative integers.

Explanation: To complete a null-words puzzle, we insert in the empty spaces those integers which make the numbers in each 'word' total zero. A 'word' is any unbroken vertical or horizontal string of numbers.

Note that there is only one correct number for each square, and each correct number can be computed, so don't guess!

One mistake and the whole puzzle blows up; we better work in pencil.

good idea!

7	3	-6						7	15	13	-9			-11	
			5							6	4	12			
		-4			6		-18	3				-5		7	
		7		7		7			-8		6	9	-7	12	4
		11												4	
	-20		8											21	-9
												9		6	
	5														15
		7											-16	9	
-3		8										.	12		
9		4	-3									-7			8
7			6	8			-9	7		-9				14	
0				-9		-7			-5						
9					-6			11		14					
-4	7		0	11	-19				-23			7	-8	-9	14

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A GAME TO PLAY

This is a game for two players, A and B. You will need a rectangle of cardboard, about 3cm wide and 30cm long. You will also need a counter and two dice (preferably different colours).

a) Make the game board as shown below:

A	- 12	- 11	- 10	.....	.....	10	11	12	B

b) If your dice are different colours, choose one of them to be negative. If they are both the same colour, mark each face of one die with a negative sign, using a crayon or coloured pencil.

c) You are now ready to begin. Decide whether A or B will go first.

The Rules

1) The counter is put on the space below 0.

2) The player take turns to throw the two dice. The total score for the throw is the scores on the two dice added together.

3) If the total score is positive, move the counter right. If it is negative, move the counter left.

Example:

Player	Scores on dice	Total Score	Result
B	6 and - 2	4	Move 4 places right to 4
A	- 6 and 1	- 5	Move five places left to - 1
B	- 3 and 3	0	Stay at - 1

4) The game is over when the counter lands on or beyond the 12 or - 12 space. If the counter lands on 12, player A wins. If it lands on - 12 player B wins.

MULTIPLICATION MATRIX

X	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4	5
- 5	25	20	15	10	5	0	- 5	- 10	- 15	- 20	- 25
- 4	20	16	12	8	4	0	- 4	- 8	- 12	- 16	- 20
- 3	15	12	9	6	3	0	- 3	- 6	- 9	- 12	- 15
- 2	10	8	6	4	2	0	- 2	- 4	- 6	- 8	- 10
- 1	5	4	3	2	1	0	- 1	- 2	- 3	- 4	- 5
0	0	0	0	0	0	0	0	0	0	0	0
1	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4	5
2	- 10	- 8	- 6	- 4	- 2	0	2	4	6	8	10
3	- 15	- 12	- 9	- 6	- 3	0	3	6	9	12	15
4	- 20	- 16	- 12	- 8	- 4	0	4	8	12	16	20
5	- 25	- 20	- 15	- 10	- 5	0	5	10	15	20	25

This grid reinforces the concept of multiplication of positive and negative numbers and the effect on the sign of the answer:

X	Negative Number	Positive number
Negative number	Positive answer	Negative answer
Positive number	Negative answer	Positive answer

NB. The same pattern exists for division.

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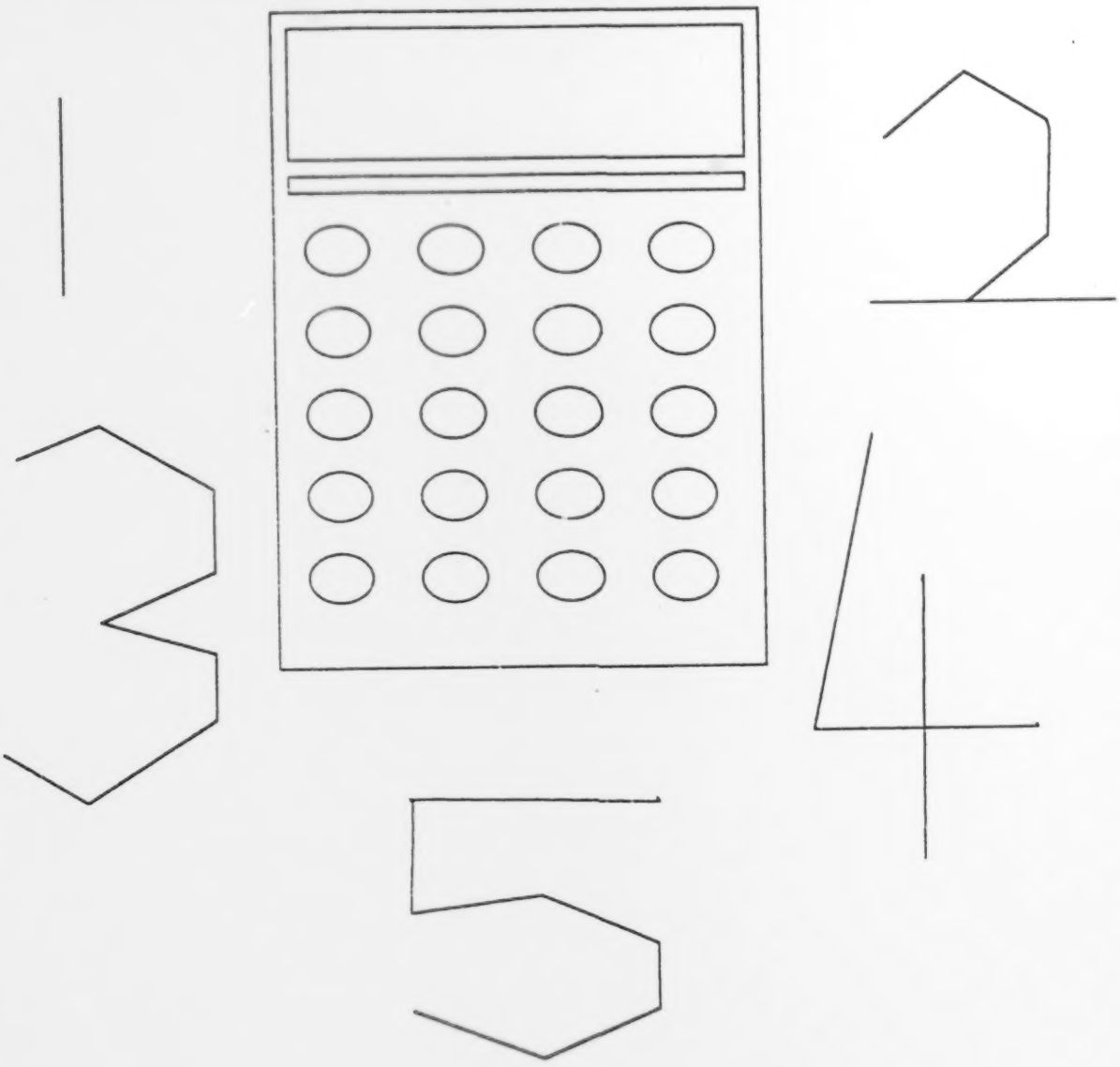
Most of these exercises involve relatively small numbers. It is anticipated that with considerable practice at the previous exercises students should be developing an understanding of all the concepts involved. They should then be able to transfer their skills to the manipulation of large numbers.

The teacher should decide a suitable point to introduce the use of a calculator as a further aid to understanding. A noticeable advantage to using the calculator is that the operation and the sign are separated. (Students will need to be taught to use the [+/-] to change the sign of the number). This division between operation and sign may help students who are still experiencing mis-understandings.

It is worth noting that for most written examinations students are allowed to use a calculator - if these are available then 'move with the times' and use one.



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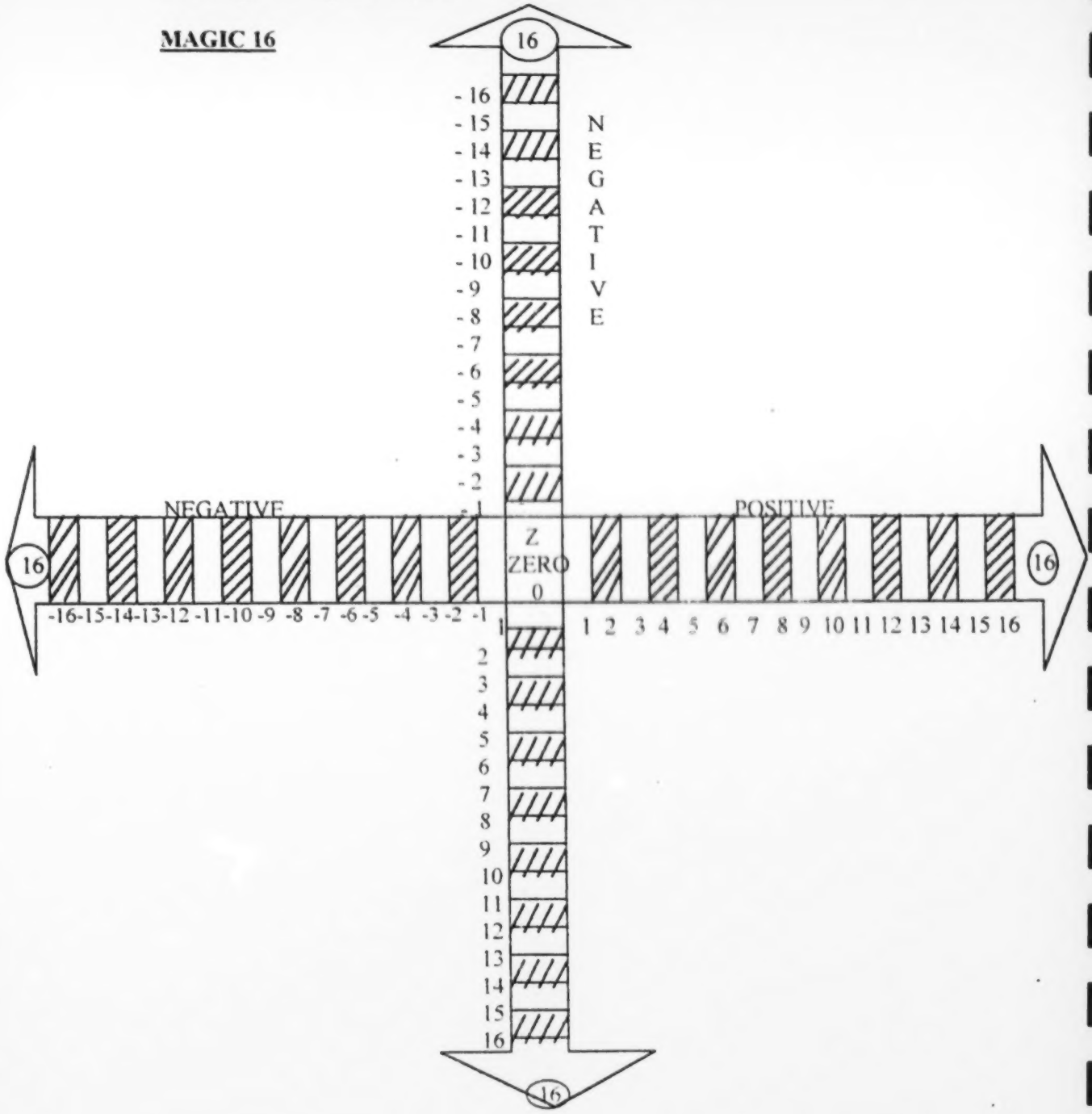
This board was devised by Mr. J. White, Petit Bordel Secondary School, as part of his Teaching Training Certificate.

MAGIC 16



The original board makes use of colour to distinguish positive and negative sections of the board.

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1) Use coloured dice to represent positive and negative. Players move horizontally or vertically from zero until they reach 'MAGIC 16'. The number of spaces moved equals the total of the two dice.

2) Let students devise their own games.

CONCLUSION

For the teachers involved in this workshop the discussions and development of ideas and exercises was very profitable. We have yet to test the success of some of these activities. It was agreed that it is important to give students a range of experiences in Mathematics and certainly for those topics which many students find difficult - Directed Numbers being high on this list. It is generally beneficial to display information and exercises on the classroom walls - thus presenting a constant visual reminder of work undertaken.

We have not discussed 'drilling' the rules for manipulating directed numbers - if students are approaching external examinations and are still experiencing difficulties then 'drilling' may be necessary. However, since students need to apply these concepts to many other topics in Mathematics, drilling the basics may be insufficient to allow further application of knowledge.

It is hoped that teachers will find this booklet a useful resource and obtain success from using the activities.

Some more able students often query why multiplication of two negative numbers result in a positive answer. A proof is given as an appendix.

Workshop Members.

Mr. W. King	Trou maca Ontario Secondary School	
Mr. S. Jocelyn	" " " "	
Mr. A. Douglas	" " " "	
Mrs. J. Dingley	" " " "	/ V.S.O.
Ms J. Jordan	Petit Bordel Secondary School	
Mr. G. Dingley	" " " "	/ V.S.O.

The workshop was led by Mr. King, who provided most of the practical activities. This booklet was collated and prepared by Mrs. Jan Dingley, VSO, North Leeward. Typing, design, layout and graphics were done by Mr. S. Jocelyn, Maths teacher, T.O.S.S.

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APPENDIX

PROOF: MULTIPLICATION OF DIRECTED NUMBERS

Mr. Winsford King, T.O.S.S.

1) To find (+4) x (-5)

$$4 \times 0 = 0$$

$$4 \times (5 + (- 5)) = 0$$

$$(4 \times 5) + (4 \times (- 5)) = 0$$

$$20 + (4 \times (-5)) = 0$$

But $20 + (- 20) = 0$

therefore $4 \times (-5) = - 20$

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2) To find (-4) x (-5)

$$- 4 \times 0 = 0$$

$$- 4 \times (5 + (- 5)) = 0$$

$$((- 4) \times 5) + ((-4) \times (-5)) = 0$$

But $(- 4) \times 5 = 20$ (Above)

therefore $((-4) \times (-5)) + (-20) = 0$

But $20 + (-20) = 0$

therefore $(-4) \times (-5) = - 20$

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